Understanding Hot-Wire Anemometry

Introduction
Hot-wire anemometry is a technique for measuring the velocity of fluids, and can be used in many different fields. A hot-wire anemometer consists of two probes with a wire stretched between them. The wire is usually made of tungsten, platinum or platinum-iridium [1]. A small, glass-coated thermistor bead is often used on constant-temperature circuit versions. Figure 1 below shows several different examples of hot-wire anemometers:

![Figure 1. Schematic of a Typical Hot-Wire Anemometer [1].](image1)

A hot-wire anemometer works as follows: an electric current is sent through the wire, causing the wire to become hot. As a fluid (typically air) flows over the device, it cools the wire, removing some of its heat energy. An energy balance equation can be used to describe this heating and cooling of the wire. This equation can then be solved to determine the velocity of the fluid flowing over the wire.

One advantage of hot-wire anemometers over other velocity measurement sensors is that they can be made very small to minimize their disturbance of the measured flow. Hot-wire anemometers are also very sensitive to rapid changes in velocity because the wire has a small time constant.

Theory and Application
Hot-wire anemometers can be operated in either constant current or constant temperature configurations. In the constant current mode there is a danger of burning out the wire if the cooling flow is too low. Likewise, if the flow is too high, the wire will not heat up sufficiently to provide good quality data [1]. For these reasons and more, most hot-wire anemometers are used in a constant temperature configuration, and we will limit our discussion to this design.

To obtain the most accurate data possible, hot-wire anemometers are typically used as part of a Wheatstone bridge configuration. An example of a constant temperature Wheatstone bridge circuit is shown in Figure 2.

![Figure 2. Constant Temperature Circuit Diagram [1].](image2)

The circuit is composed of two known fixed resistors $R_1$ and $R_2$ and a third variable resistor $R_v$. The hot-wire probe is the fourth resistor $R_w$ that completes the bridge. The bridge is balanced when $R_1/R_w = R_2/R_3$, resulting in...
no voltage difference or “error voltage” between points 1 and 2.

The constant temperature circuit takes advantage of the fact that the wire resistance \( R_w \) is a function of temperature. It works as follows: when the wire temperature and resistance are at some initial operating point, the variable resistor \( R_3 \) can be adjusted to bring the bridge into balance. As the air speed over the wire is increased or decreased, the temperature of the wire changes, and so does the resistance. This effect causes the bridge to become unbalanced, resulting in a voltage difference between points 1 and 2. The amplifier detects this difference. It adjusts the feedback current accordingly to keep the wire temperature and resistance constant, and thus re-balances the bridge. These changes in current can be measured and used to calculate the flow velocity over the wire.

To understand the relationship between the current and the flow velocity, it is necessary to solve the heat balance equation for the wire filament [2]. To keep the analysis simple, only the steady-state conditions will be considered. The general heat balance equation for the wire filament is:

\[ H_g = H_T + H_A \]  

(1)

For steady state conditions there is no heat accumulation \( H_A \) in the wire so this term goes to zero. The heat generation, \( H_g \) by joule heating is a function of the electrical power input to the wire. It is defined as:

\[ H_g = I^2 R_w \]  

(2)

Where,

\[ I = \text{current through the circuit} \]

\[ R_w = \text{wire resistance at temperature } \Theta_w \]

To determine \( H_m \), the value of heat transferred to the fluid, it is necessary to relate the wire resistance and general heat transfer equations. The wire resistance as a function of temperature can be described by the following series expression:

\[ R_w = R_o [1 + C(q_w - q_0) + C_2(q_w - q_0)^2 + ...] \]  

(3)

Where,

\[ R_o = \text{wire resistance at a given initial reference temperature} \]

\[ \Theta_0 = \text{initial wire reference temperature} \]

\[ C = \text{temperature coefficient of resistivity} \]

Disregarding the higher order terms, and applying the boundary condition \( R_s = R_g \) when \( \Theta_o = \Theta_g \) the following expression results:

\[ \Delta q = \frac{R_w - R_g}{R_o C} \]  

(4)

Where,

\[ R_g = \text{wire resistance when the wire temperature equals that of the fluid to be measured} \]

\[ \Delta \Theta = \text{temperature difference between the wire and the fluid (} \Theta_w - \Theta_g \text{).} \]

A useful empirical heat transfer equation that describes the heat transfer for a fluid passing over an infinite rod is as follows:

\[ Nu = 0.42 Pr^{0.2} + 0.57 Pr^{0.33} + Re^{0.50} \]  

(5)

Where,

\[ Nu = \frac{hd}{k} \]  

(Nusselt number)

\[ Pr = \frac{mC_p}{k} \]  

(Prandtl number)

\[ Re = \frac{r Ud}{m} \]  

(Reynolds number)

and where,

\[ h = \text{convective heat transfer coefficient} \]

\[ d = \text{characteristic length (wire diameter in this case)} \]

\[ k = \text{fluid thermal conductivity} \]

\[ \mu = \text{dynamic viscosity of the gas} \]

\[ \rho = \text{gas density} \]

\[ Cp = \text{specific heat of the gas at constant pressure} \]
U = velocity of the flow

Disregarding radiation and conduction through the wire, and assuming convection only, we have the following expression for HT:

\[ H_T = hA_s \Delta q \]  \hspace{1cm} (6)

Where,

\( A_s = \) Surface area of the wire exposed to the fluid flow

Substituting Equations (4) and (5) into (6), and adding some algebraic manipulation, the following expression results:

\[ H_T = (R_w - R_g)(X + Y\sqrt{U}) \]  \hspace{1cm} (7)

Where,

\[ X = \frac{0.42kA_s}{R_oCd} \left( \frac{mC_p}{k} \right)^{0.2} \]  \hspace{1cm} (8)

And,

\[ Y = \frac{0.57kA_s}{R_oCd} \left( \frac{mC_p}{k} \right)^{0.33} \left( \frac{r d}{m} \right)^{0.5} \]  \hspace{1cm} (9)

If we define a resistance ratio as \( R = R_u/R_g \) and substitute Equations (2) and (7) into Equation (1) the following expression results:

\[ I^2 = A + B\sqrt{U} \]  \hspace{1cm} (10)

For a given wire, the value of \( R \) is constant so that Equation (10) can be reduced to:

\[ I^2 = \left( \frac{R - 1}{R} \right)(X + Y\sqrt{U}) \]  \hspace{1cm} (11)

This Equation is referred to as King’s Law. To calibrate the hot-wire anemometer, the second power of the measured values for the current \( I \) are plotted vs. the square root of corresponding known velocities, \( \sqrt{U} \). A best-fit straight line can be fit to the data, and hence the values of the constants A and B of Equation (11) can be determined.

It is important to note that bulk analysis oversimplifies what is actually happening in the hot-wire anemometer. A complete analysis would need to consider, among other things, axial heat conduction in the wire, heat loss at the wire attachment points on the probe, aero-elastic behavior of the wire, and the dynamic system response for both the heated wire and the measurement circuitry.

Finally, there are a number of measurement errors that must be accounted for in the calibration and use of hot-wire anemometers. These include, but are not limited to, the following [3]:

1. Calibration measurement errors: Errors in measuring the calibration flow parameters and hot wire voltages.

2. Calibration equation errors: Errors due to the fitting of a calibration equation, as well as the solution of the calibration equation and lookup table.

3. Calibration drift errors: Errors caused by variations in calibration over time and due to switching the feedback circuitry on and off, as well as by probe contamination.

4. Approximation errors: Errors caused by assumptions about the flow field that are used to solve the calibration equations.

5. High frequency errors: Errors caused by the change in hot wire behavior at high frequency.

6. Spatial resolution errors: Errors caused by spatial averaging of the flow field.

7. Disturbance errors: Errors caused by the probe interfering with the flow field.

Also, at low flows, the effects of natural convection introduce errors in the calibration. Even though the velocity in the calibration wind tunnel is set to zero, the heated wire still transfers heat energy to the environment due to natural convection buoyancy effects.

**Conclusion**

Hot-wire anemometers are excellent tools for measuring the flow velocity of gases and inert liquids. Hot-wire
ATS’ laboratory grade bench top wind tunnels are designed with polynomial shapes to provide highly uniform flow for accurate characterization of components, circuit boards and cooling devices such as heat sinks, heat exchangers and cold plates. Each features ports to accommodate a variety of probes including thermocouples, Pitot tubes and temperature and velocity sensors. Each bench top wind tunnel is designed to be lightweight and compact and feature Plexiglas® test sections for clear views of the specimens and flow visualization.

- **BWT-100**
  - Produces flow velocities from 0 to 2 m/s (400 ft/min)
  - Test Section Dimension: 21.6 cm x 25.4 cm x 2.5 cm (8½” x 10” x 1”)
- **BWT-104**
  - Produces flow velocities from 0 to 6 m/s (1200 ft/min)
  - Test Section Dimensions: 50.8 cm x 17.25 cm x 10 cm (20” x 17.25” x 4”)
- **WTC-100**
  - Measures temperatures from -10°C to 150°C (±1°C)
  - Capable of controlling velocities from up to 50 m/s (10,000 ft/min) depending on the fan tray
  - Features a user friendly, labVIEW based, application software
anemometers offer excellent spatial resolution with minimal flow disturbance, as well as excellent dynamic response to rapid changes in flow velocity. The calibration of hot-wire anemometers requires care, and special attention must be paid to the multiple error sources that can affect accuracy. But, these devices continue to be a standard tool for engineers who need to measure the velocity of fluids.

References:

